

# The Sheffer Box



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UC Berkeley Logic Colloquium, 11/21/2014

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## M. Davis, American Logic in the 1920s (BSL 1995)

The writings of American logicians during the 1920s [form] a period of important beginnings and remarkable insights as well as of confused gropings . . .

Studies like this . . . can have the effect of bolstering a sense of our superiority to our logical forbears. But this would be a serious mistake. The lessons to be learned are rather that the development of the outlook on our subject that today we take for granted was attained only with great difficulty.

## The Sheffer Box

- 1964 Sheffer dies, having asked B.S. Dreben to order his materials in Widener Library for the Harvard Archives
- 1970 Dreben and H.S. Leonard archive 50 boxes at Harvard
- 1999 B.S. Dreben dies.
- 2012 Discovery of the Sheffer Box
- 2013 Sheffer Box placed in the Harvard Archives

## Main Themes

1. American logic in the 1920s emerged from American logic, philosophy, mathematics in the 1900's, 1910s
2. Sheffer a pioneer in model theory, "invariance" account of logical form, structural approach to logic
3. Sheffer an influential conduit for math/logic in the 1910's-'20s (Russell, Langford, Zermelo, Tarski)
4. Sheffer and Pragmatism: Neutral Monism, Russell, Wittgenstein, C.I. Lewis, Quine, S.K. Langer
5. Sheffer "wandered too far from the formalist pole" (Quine to Russell, 1935)

## H. M. Sheffer, 1883-1964

Usually remembered for:

1. Sheffer Strokes of joint denial and alternate denial, a functionally adequate set of Boolean Connectives for truth-functional logic.
2. 2nd ed. Intro, *Principia Mathematica*, Russell: Sheffer's the "most definite improvement" in logic since 1st. ed; recommends a "complete rewriting" of *Principia* by HMS using his "new, very powerful method".
3. HMS 1926 review of PM 2nd ed.: "Just as the proof of certain theories in metaphysics is made difficult, if not hopeless, because of the 'egocentric' predicament, so the attempt to formulate the foundations of logic is rendered arduous by a corresponding '**logocentric**' predicament: in order to give an account of logic, we must presuppose and employ logic."

## Sheffer I: 1883-1910

- 1883 Born in Ukraine; c. 1892 moves to Boston; 1902 Graduates Boston Latin School
- 1905 Harvard BA, studies with James, Royce, Huntington
- 1906-1910 Assistant to Royce; 1907 Harvard MA
- 1908 Harvard Ph.D “A Program of Philosophy, Based on Modern Logic” (signed by Royce, Huntington, and Holt)
- 1908-9 post-doc at Harvard
- 1910-11 travels on a Sheldon Fellowship to Cambridge, Paris, Turin, Göttingen, Jena. Works with Russell, meets Peano, Padoa, Burali-Forti, Frege.

## Sheffer II: 1911-1916

- 1911-1912 University of Washington, Seattle
- 1912-1913 Cornell University
- 1913-1914 University of Minnesota
- 1914-1915 University of Missouri, Columbia  
(Russell at Harvard spring 1914 lecturing on Wittgenstein's "Notes on Logic")
- 1915-1916 CCNY  
(Post graduates from CCNY 1917, at Columbia 1917-1920)
- 1916 University of Michigan

## Sheffer III: 1917-1926

- 1917-1926 Instructor at Harvard (death of Royce in September 1916)
- 1927 Assistant Professor of Philosophy, Harvard
- 1938 Promoted to Full Professor
- 1940 University of Chicago  
(Russell, Carnap, and Tarski at Harvard)
- 1952 Retires from Harvard

## Sheffer's Works through 1922

1. "A Program of Philosophy, Based on Modern Logic", Harvard PhD, 1908 (signed by Holt, Huntington, Royce)
2. "Ineffable Philosophies", JPhil 1909
3. Reviews, Abstracts of Talks (Bull. AMS)
4. "A Set of Five Independent Postulates for Boolean Algebras, with Application to Logical Constants", Trans.Amer.Math.Soc.1913
5. "The General Theory of Notational Relativity", mimeographed edition, 1921. 61pp.
6. Marvin Farber's Notes of Phil 8, spring 1922

## Scholarship

- M. Scanlan (1991), "Who Were the American Postulate Theorists?", *JSL* 56,3: 981-1002
- M. Scanlan (2000), "The Known and Unknown H.M. Sheffer", *Trans.Peirce Soc.* 36,2: 193-224
- M. Scanlan (2010), "Sheffer's Criticism of Royce's Theory of Order", *Trans.Peirce Soc.*, 46,2: 178-201
- A. Urquhart (2012), "Henry M. Sheffer and Notational Relativity", *Hist.Phil.Logic* 33: 33-47



## Sheffer 1908: Philosophical Themes

- **Logic as the Essence of Philosophy** (Royce, Russell):  
Rejection of “ineffable” philosophies (including that of Royce’s “will acts”, Münsterberg, Bergson...).  
All philosophy consists of propositions
- **General Theory of Order** (Royce, Huntington):  
Model theoretic realism (no “free creation”)  
“Logicism”: Postulate Theory  
Theory of the actual infinite to be included
- **Pragmatism, Realism, “Neutral” Monism** (James, Holt):  
Philosophy reflects temperament; Psychology Relevant  
No *a priori* necessities in logic  
Consciousness to be studied scientifically

## Contrast with C.I. Lewis's Dissertation

Lewis (1910): "The Place of Intuition in Knowledge"

Aim: To reconcile James and Royce

Idea: Chart a middle course between

- "Intuitionism": empiricism, phenomenalism: some things are "given" in experience and not constructed (James)
- "Actionism": Rationalism, realism, voluntarism: the World order is constructed by the actions of mind and the world is an infinite system (Dedekind) that we read with signs (Royce)

## Sheffer 1908: Main Themes

- **The Primacy of Logic:** Generalized Postulate Theory: Philosophy as discovery of postulates for selected sub-manifolds of experience; all manifolds are definable.
- **The Neutral Realm:** the known world is built up from this by selectivity. (Pluralism of systems, not of types of “Being”.)
- **Discovery/Consistency:** “We are really seeing, not consistence, but existence. What we want to know about our ‘postulates’ is whether they are neutral entities actually found in the neutral realm..or whether they are only ‘free creations’, that is, a string of words (*Mengenlehre*; Mind; Frege)

## Sheffer 1908

### References to Russell:

- Russell, *Principles of Mathematics* (1903)
- Russell, "On the Nature of Truth" (1906/7)
- Endorses a version of Russell's MRTJ theory of truth for *Principia*, but no reference made to "On Denoting" (1905):  
HMS doesn't contemplate eliminating propositions!  
Nor was he working with any notion of fact.  
He does grasp the idea of *propositional function*.

## Sheffer 1908

- Sheffer analyzes correspondence *postulationally*.
- Truth is given by correspondence of propositions given by a postulate set (selected from the neutral realm) to a model, i.e., “truth in a system structure”. Belief is analyzed in terms of selected manifolds from the “neutral” realm.
- Outstanding problem:  
What does “truth in a system structure” mean *in general*?

## Sheffer 1908: Neutral Monism

- “Types” of objects must be distinguished (Mental, Physical, Mathematical) according to distinct principles of selection (definability)
- “Ultimately our problem is not consistency, but selectivity” from within the neutral realm.
- MRTJ: Let us consider the total situation of what we may call “A’s perception of the ink-well”. We are asking for the relation between entities  $a, b, c$ , etc. of a given class  $\mathbf{K}(a, b, c)$  and that class  $\mathbf{K}$  itself.

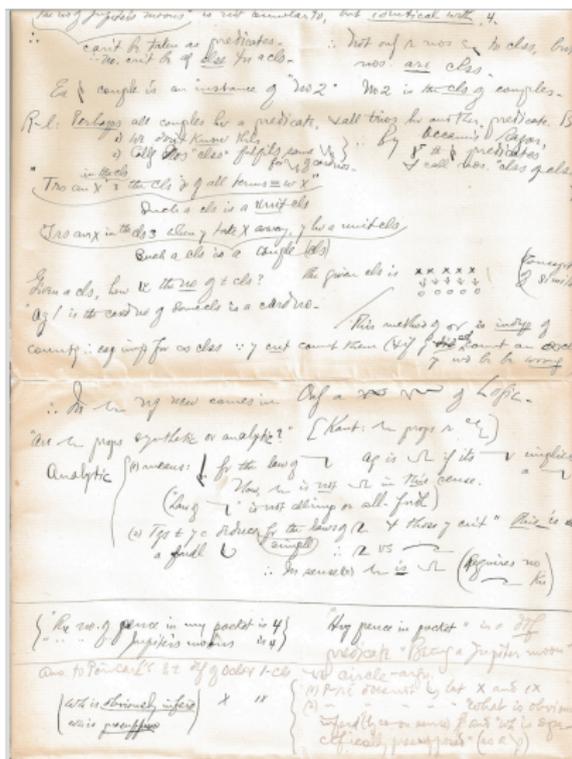
## Sheffer 1908: Appendix

- Sheffer refutes Royce's 1905 reduction of asymmetric relations to symmetric relations in the O-System
- Royce selected an arbitrary point of origin  $y$  for the ordering  $\text{---} <_y$
- But this needs an axiom (Scanlan 2010)
- "Once Asymmetric, Always Asymmetric"
- Symmetric relations: Develop Royce's suggestion of looking for new (number-theoretic) models of algebras
- Notion of a "Boolean" algebra named

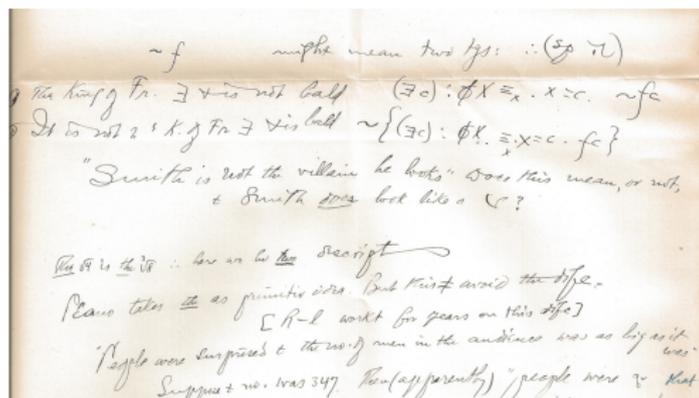
## 1910-11: Cambridge, Paris, Göttingen, Jena, Turin

- HMS attended Russell's first Cambridge lectures, Fall 1910 (*Principia* had just appeared)
- Two courses, "Symbolic Logic", "Philosophy of Math"
- Sheffer's are the only extant copy of notes
- Taken down in idiosyncratic shorthand
- To be published (eds. B. Linsky, J. Levine) with other student notes from 1910 (G.E. Moore) and 1914 (T.S. Eliot, V. Lenzen)

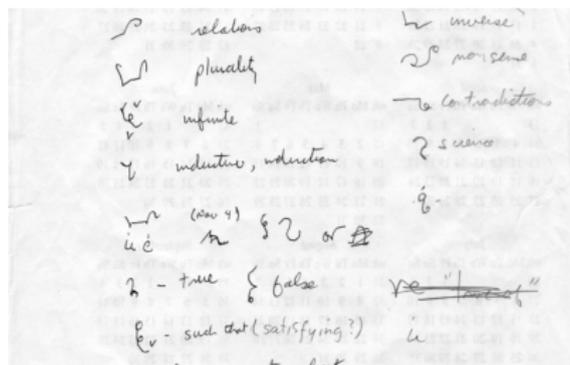
# Sheffer's notes of Russell's 1910 Cambridge Lectures



# Sheffer's notes of Russell's 1910 Cambridge Lectures



# Linsky's Rosetta Stone



## Sheffer 1913: The Stroke

- The stroke is an application of the Boolean structure of symmetric relations to *Principia*.
- Sheffer was looking for a general method of proving logical consistency that would be independent of any particular formalization of logic, and emerge by a general method internal to logic, a kind of algebra of all logics or postulate systems.
- Idea: Generality as constancy of form (invariance) under all permutations of the domain and under all alterations of universe of discourse (i.e., reinterpretations of a set of postulates), as well as a theory of “notational” relativity taking into account structural invariances among notational systems.

## Wittgenstein on Sheffer 1930

Wittgenstein used a generalized version of the Sheffer Stroke for his (1913-1918) *Tractatus* account of logic. He came to regret his way of using it.

TS 213 §134:

You could see a definition without seeing its point.

Of course Sheffer's discovery is not the discovery of the definition

$$\sim p \ \& \ \sim q \ = \ p \ | \ q.$$

It corresponds to the discovery that  $x^2 + ax + \frac{a^2}{4}$  is a special case of  $a^2 + 2ab + b^2$ .

We don't see that something can be looked at in a certain way until it has been so looked at. We don't see that an aspect is possible until it is there.

# General Theory of Notational Relativity

- 1916-18: Sheffer attempts to carry out his mathematical program.
- Several abstracts of mathematical talks sound very grandiose.
- 1919 to Russell:  
I have just finished a manuscript under great pressure of time. I fear it must be filled with paradoxes. Please read it. Then send it back to me.

# General Theory of Notational Relativity 1921

HMS to Russell, 1919:

This manuscript is a series of *Gedankenexperimente*.

Preface:

The following pages present, in outline, a new *method* in mathematical logic. In a volume entitled *Analytic Knowledge*, which the writer hopes to publish in the near future, this method—which may be characterized as a sort of Prolegomenon to Every Future Postulate Set—is developed in detail, and is then applied to the solution of a number of fundamental problems in logic, mathematics, and *Mengenlehre*.

# General Theory of Notational Relativity 1921

Circulated in manuscript, 1921, in order to obtain promotion at Harvard. (This only happened in 1927, however).

- Focus on structural patterns within order types.
- Works discretely for the finite case, aiming to generalize.
- Isolates the first few Bell numbers (number of partitions of a set/number of equivalence relations on a set): 1, 1, 2, 5, 15, 52
- Obtains results identical with Tarski's 1966 lecture "What are logical notions?" (Urquhart 2012)

## Tarski 1966/1986

Logical notions are precisely those that are invariant under all permutations of the universe.

If we . . . consider binary relations, a simple argument shows that there are only four binary relations which are logical in this sense: the universal relation which always holds between any two objects, the empty relation which never holds, the identity relation which holds only between "two" objects when they are identical, and its opposite, the diversity relation. So the universal relation, the empty relation, identity, and diversity—these are the only logical binary relations between individuals.

## Tarski 1966/1986

This is interesting because just these four relations were introduced and discussed in the theory of relations by Peirce, Schröder, and other logicians of the nineteenth century. If you consider ternary relations, quaternary relations, and so on, the situation is similar: for each of these you will have a small finite number of logical relations.

## Tarski 1966/1986

The situation becomes a little more interesting if you go to the next level, and consider classes of classes. Instead of saying 'classes of classes' we can say 'properties of classes', and ask: What are the properties of classes which are logical? The answer is again simple, even though it is quite difficult to formulate in a precise way. It turns out that the only properties of classes (of individuals) which are logical are properties concerning the number of elements in these classes. That a class consists of three elements, or four elements . . . that it is finite, or infinite—these are logical notions, and are essentially the only logical notions on this level. Now in the light of our suggestion it turns out that our logic is even less than a logic of extension, it is a logic of number, of numerical relations.

## Sheffer on “System Functions”

Russell has introduced the important generalization of the concept of proposition to that of propositional function. We find it useful to coin analogous terms for certain analogous generalizations.

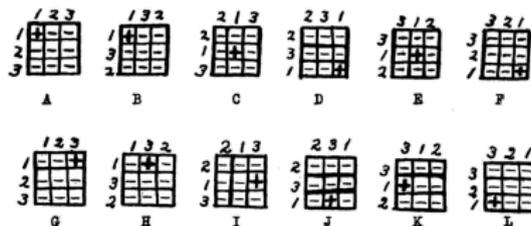
When, in the current literature, the relation “sphere-inclusion” is replaced by  $R$  we have no right to call  $R$  a relation.  $R$  is, and should be called, a relational function. Similarly, when the class of “solid spheres” is replaced by  $K$ , we must call  $K$  a class function.

## Analysis of “Postulate”

When the class of “solid spheres” is replaced by  $K$ , and the relation of “sphere-inclusion” is replaced by  $R$ , the Huntingtonian language becomes the Huntingtonian language function  $[K, R]$ . The name “language function” is conveniently replaced by the name base.  $[K, R]$  is therefore the Huntingtonian base. The propositions that we have called the Huntingtonian assumptions become now the propositional functions that we may call the Huntingtonian assumptional functions. However, the name “assumptional functions” is conveniently replaced by the name “postulates”.

## Dyadic Grafs, " $K^3R_2$ "

Consider the family of all structures consisting of a binary relation  $R$  on a three element set, where  $R$  contains exactly one ordered pair  $\langle x, y \rangle$  with 1)  $x = y$ , and 2)  $x \neq y$ .



# Isotropy

Let  $F$  be a family of relational structures  $\langle K, R \rangle$  of the form  $K^n R_m$ . Assume  $K$  is the standard  $n$ -element set  $\{1, \dots, n\}$ .

Let  $\alpha$  be a permutation on  $K$ ; for  $x \in K$ , write  $x\alpha$  for the image of  $x$  under  $\alpha$ . A permutation  $\alpha$  acts on the family  $F$ .

Define:

$\langle x_1, \dots, x_m \rangle \alpha : \langle x_1 \alpha, \dots, x_m \alpha \rangle$  and

$R\alpha = \{ \langle x_1, \dots, x_m \rangle \alpha : \langle x_1, \dots, x_m \rangle \in R \}$

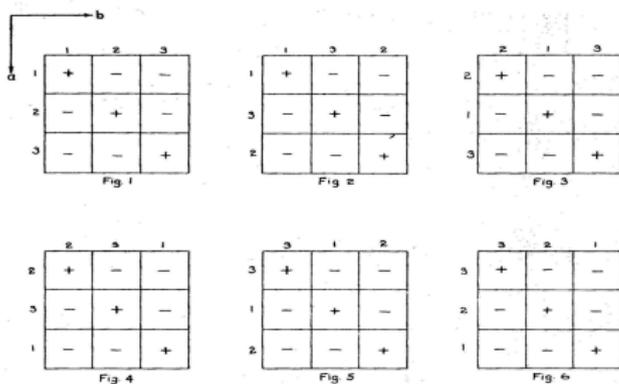
A family of structures is *isotropic* if it consists of a single structure  $\langle K, R \rangle$ .

# Isotropy

We use the term “permutation” rather than “order”, because, in the case of a non-finite  $K$ , the elements may be taken with different permutations within one order type. Thus, 1, 2, 3, 4, 5, 6, . . . and 2, 1, 4, 3, 6, 5, . . . , are different permutations of a countable set of elements within the one order type  $\omega$  . . . Hence, postulationally, it does not matter whether the three elements of our system function on  $K^3R_2$  are taken with permutation 1, 2, 3, as in Fig. A, or with any other of the possible  $3!$  permutations.

# Isotropy

KZ (“zero”): System function furnishes “zero” information about postulational distinctions among elements.



# Superpostulates

A **Superpostulate** is an implication of the form:

$$\forall a_1 \dots \forall a_m [I(a_1 \dots a_m) \rightarrow \pm Ra_1 \dots a_m],$$

where

$I(a_1 \dots a_m)$  is a conjunction

stating whether  $a_i = a_j$  or  $a_i \neq a_j$ , for  $i \neq j$ .

It is a “super postulate” because it encompasses two distinct postulates, depending on the choice for  $\pm$

. It is atomic because the consequent of the implication is an atomic formula or its negation.

# Stratigraphy, Permutativity

- Stratigraphy: a theory of relational structures that are partially, but not fully symmetric: remove one element and classify.
- Permutativity: Structures in a stratigrafic family are divided into isotropic elements, together with “stratigrafic elements”. Classify these elements as to their behavior under permutations.
- This project is not brought to fruition.

## Foundations of Physics?

- Sheffer's idea (with Russell's blessing) was to apply the technique to Einstein's theory of relativity.
- This is mentioned but not developed in the manuscript. Russell refers to "neutral monism" in ch. 1 of *The Analysis of Matter* (1927).
- An inscribed copy of P. Frank's Harvard Physics lectures was found in the Sheffer Box.

## Post to Sheffer Nov.-Dec. 1921

Your title suggests that it ought to have an extensive bearing on my own research . . . and made me want to send you a brief summary of my present research program (which suffered a complete reversal in the last five weeks).

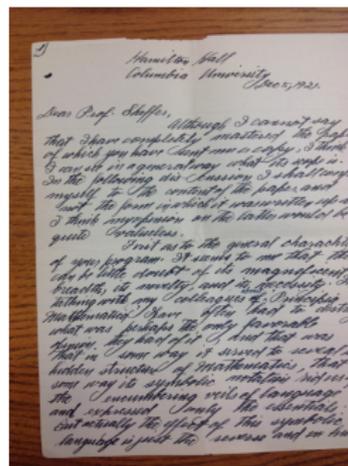
## Post to Sheffer Nov.-Dec. 1921

- As to the general character of your program. It seems to me that there can be little doubt of its magnificent breadth, its novelty, and its necessity.
- In talking with my colleagues about *Principia* I have often had to describe what was perhaps the only favorable opinion they had of it, . . . that in some way it served to reveal the hidden structure of mathematics, that in some way its symbolic notation rid us of the encumbering veils of language and expressed only the essentials. But actually the effect of this symbolic language is just the reverse. . .
- It is just this clarification, this revealing-of-the-hidden-structure-of-things that you propose to do. . .

## Post to Sheffer Nov.-Dec. 1921

But the themes are insufficiently developed. . . and it would be necessary to go into great detail into the relations between the formal and informal and make sure that in giving this new development we haven't simply been blind to the host of informal and not-consciously-apprehended ideas that have crept up with the simplification of the formal development.

# Post to Sheffer Dec. 1921



## HMS to Russell Aug. 31, 1923

... by means of the protoanalytic concept of tropicity we free ourselves from a vast number of merely 'linguistic' problems in dealing with 'reality'... But even tropicity is still too 'human'.

## Urquhart (2012)

Unlike Sheffer . . . Post quickly came to realize that his plans [for a general treatment of the decision problem] were unfeasible. Realizing that a diagonal argument would prove the existence of unsolvable problems, assuming the complete generality of his notion of formal system, he was led to anticipate the work of Gödel, Church and Turing by fifteen years . . . Unlike Post, Sheffer never seems to have grasped clearly that what he was trying to do was simply impossible.

## Urquart (2012)

Why did Langford [1926,7] succeed where Sheffer largely failed? Other than his well known early paper on the axiomatics of Boolean algebras, Sheffer's only substantial logical result is the work on isotropy . . . a genuine but rather minor contribution [to the theory of finite unlabelled groups]. The reason for Langford's success is perhaps that unlike Sheffer, he was not sidetracked into dreams of a grand project, instead concentrating on concrete results about specific systems. Sheffer's most lasting contribution to logic, other than the famous stroke, may have been the early inspiration that he provided to Langford.

## Phil 8 spring 1922

However. . .

- The Sheffer Box also contains notes taken down by (legible) hand by Marvin Farber, who would later become Professor of Philosophy at Buffalo, a phenomenologist.
- These were taken to Germany in 1923 by Farber, lectured on, and passed around, along with the mimeograph of “The General Theory of Notational Relativity”.
- In particular, there was an interaction with Zermelo.

## Phil 8 spring 1922

A grand tour through contemporary logic.

- Huntingtonian postulate theory, applied to logic. Algebraic theories of containment in classes of postulate systematizations. Models (Interpretations) and methods. Independence, intertranslatability vs. completeness for deductive aspect in relation to interpretations. Literature lacking a method for model construction, for notion of completeness of characterizing logics.
- *Principia*, Russell/Lewis debate over strict implication, modal logic; multi-valued logics
- Proposes quantification over all possible universes of discourse, postulational approach to meaning of “implies”.

## Phil 8 spring 1922

- Set Theory, Foundations of Logic and Mathematics, Paradoxes
- “Queer” Logics; the theory of types, the Axiom of Choice.
- General questions of inductive logic.

Duhem's holism:

Do the postulates uniquely determine their models in experience? Note that a dissonant experience can lead either to a revision of the postulates or the dismissal of the experience as irrelevant (Galileo vs. the Church)

# HMS Response to C.I. Lewis on “Strict Implication”

Sheffer:

- Better to drop word “implies”, better psychologically.
- Each use distinguishes a kind of “implies”.
- Write down instead conditions (necessary and sufficient) on the relation in question using *Principia* notation.
- Sheffer writes down 8 different conditions on (postulates for) the relation.

## HMS response to C.I. Lewis

- Condition for the weakest relation: “giving”:  
 $\phi x$ :  $x$  is a governor of Mass. between 1900 and 1915.  
 $\psi x$ :  $x$  was elected for a term of one year.
- Be wary of supposed sufficient conditions.  
Necessary condition in some cases: holds in all universes of discourse. For “giving”: not true in every possible universe.
- Can you say false for every  $x$  for every conceivable universe?

# HMS response to C.I. Lewis

Modal theory:

$(U)$ : “for all universes of discourse”

$[U]$ : “for some universe of discourse”

$(U)(x)$ :  $\neg[\phi x \bullet \neg\psi x]$  in every universe of discourse

$[U](x)$ :  $\neg[\phi x \bullet \neg\psi x]$  is false in some universe of discourse

8 cases of implication, 4 are pseudo implication.

## HMS response to C.I. Lewis

- If you could reduce “involving” to  $\phi x$  in atoms, and  $\psi x$  in atoms, would it not then seem likely that you would then have case of “comprising”?
- And that we have here in the molar level mere difference of form?
- Conclusion of Dr. Sheffer: Russell’s propositional function with material implication has symbolic generality, but it’s not deductive or mathematical implication.

## HMS response to C.I. Lewis

- Object to Russell's definition [of implication], but not from point of view of getting neat theorems, as geometry does. But you are fooling yourself if you think it *is* deductive implication.
- Lewis could get to conditions [such as those formulated above] if he went to implication between propositional functions; but he only deals with propositional logic and justly criticizes Russell.
- But Lewis too hasn't captured deductive mathematical implication.

## HMS response to C.I. Lewis

- In *Principia*, a set of postulates for implication of propositional functions and propositions.
- Set of postulates for “involving” alone—certain difficulties.
- Ask a prior question: Is it possible? Can you work out postulates for deductive implication in general?
- It can't be done.

# Phil 8 spring 1922

Boolean Algebras introduced.

Question: can you interpret them in terms of propositions? Yes.

Truth tables presented as an interpretation of Frege's notion of the True:

*will work out like diagram . o.k.*

a 4<sup>th</sup> interp., similar to the three, but not the same. In terms of a new concept. Frege has simple use of notation "truth-values" (true or false). Truth-value of a true prop. is t, & of a false prop. is false. *should be have one term*

$K \equiv$  (cls of prop. truth-values)

$C =$  (cls whose only member is the truth value truth)

$a' =$  opposite t.v. of a

$a * b =$  disjunctive t.v. of (a, b)

$\therefore a * b =$  disj. t.v. of (opp. t.v. of a & of b)

	0	1	0	1
prop. a	t	t	f	f
prop. b	t	f	t	f
$a * b$	t	f	f	f

## Phil 8 spring 1922

Sheffer ends his course with open problems.

- Are the principles of logic incomplete, and do they need, as a principle, Zermelo's [Axiom of Choice]?
- Is it a theory? Or false? Or valid only in countable form?
- Another phase of problem of foundations of logic. Can we have non-Euclidean logics? Strange logics?
- Queer logics: queer as you please and still keep your feet on ground, and logics which are not logics.
- No end of unsolved problems in this field!

# Farber

- Farber went to Freiburg, Berlin, and Göttingen with these notes and the “General Theory of Notational Relativity”.
- He lectured on Sheffer’s work at Freiburg.
- Wrote home to Sheffer about the scene:  
“Becker stayed up one whole night to copy down your manuscript. . . .”

## Farber to Sheffer

Under separate cover I am sending you a *Vorlesung* I recently gave in the University here. The following week I was requested to expound once more, which I did informally. Zermelo was present, and spoke of your "*leistungen*" as being a "*grosser Fortschritt*".

- Farber and Zermelo considered writing a logic book together soon after. But the project never came to fruition.
- There is a draft in Buffalo archives.
- Gregory Taylor (2008, 2009) discusses Zermelo's conception of "the general form of proposition" in terms of invariance.

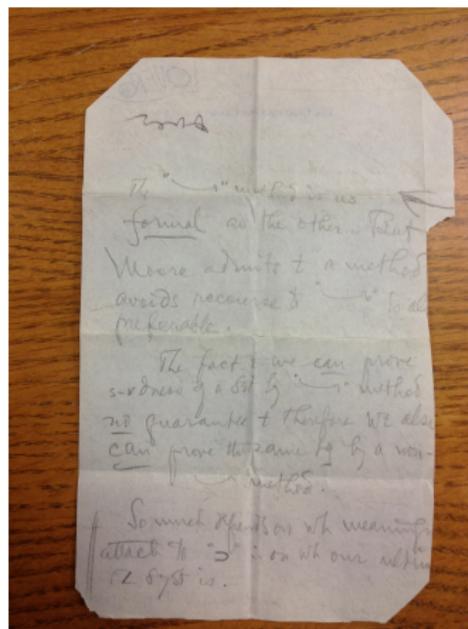
# Archives

Sheffer's Papers: an “archival wilderness” more like “archaeology” than ordinary research (Scanlan 2000)

- Harvard Archives, 50 boxes (now 51)
- Cut up slips, annotated by hand, arranged
- Decks of notes, bound together
- Sheffer's student notes, annotated, mixed authorship
- Student notes of Sheffer's seminars.
- Idiosyncratic shorthand of Sheffer.
- Correspondence

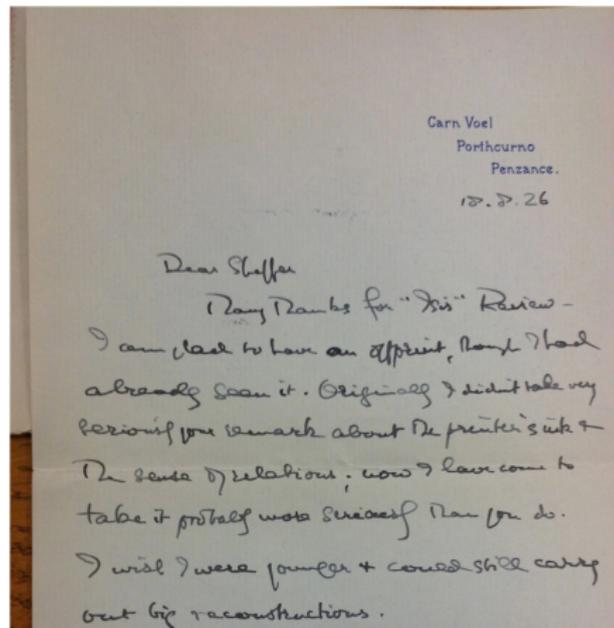
## Archives

Notes on a conversation with E.H. Moore, Oct. 12, 1912:  
 "So much turns on the meanings attached to " $\supset$ ",  
 therefore, on what our ultimate [logical] system is.



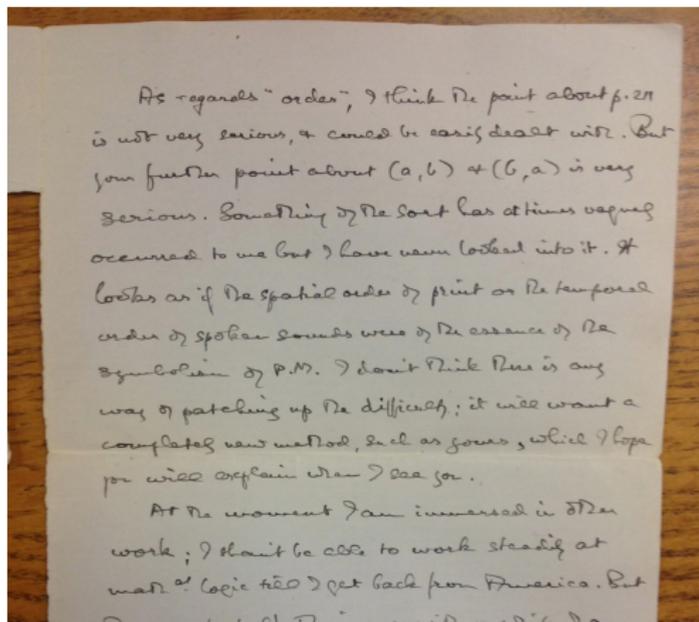
## Archives: Russell to Sheffer Aug. 18, 1926

Many thanks for the "Isis" review. . . . Originally I didn't take very seriously your remark about the printer's ink and the sense of relations; now I have come to take it probably more seriously than you do . . .



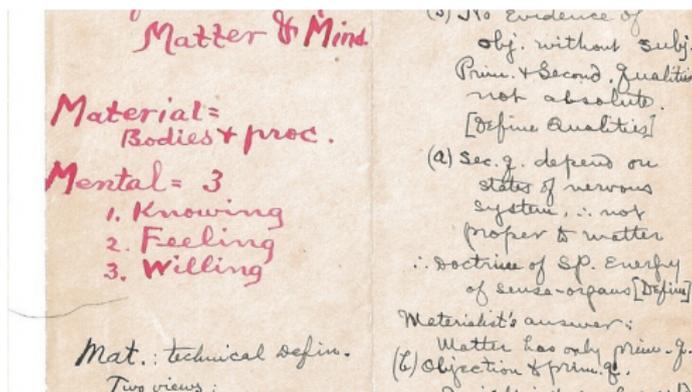
## Archives: Russell to Sheffer

... your further point about  $(a,b)$  &  $(b,a)$  is very serious. Something of the sort has at times vaguely occurred to me, but I have never looked into it. It looks as if the spatial order of print or the temporal order of spoken sounds were of the essence of the symbolism of PM.



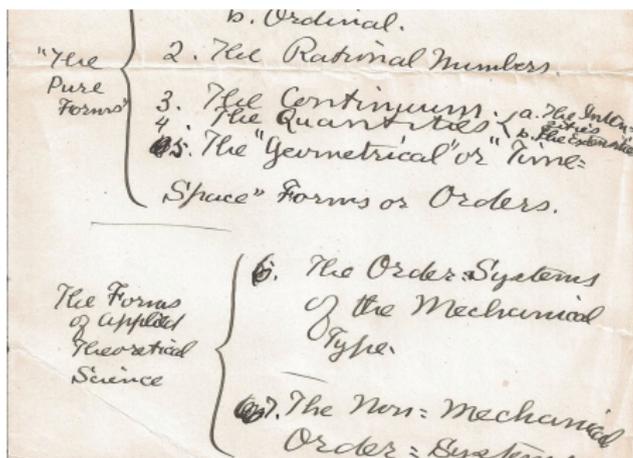
# Archives

## Notes of William James's lectures (?)



# Archives

Phil 15, Order Types as Actually Present In the Exact Sciences, for Working Out, 1910-11; Royce's hand?



## Meeting with Russell, fall 1910: the Deduction Illusion

Two kinds of individuals: (1) things (2) concepts.

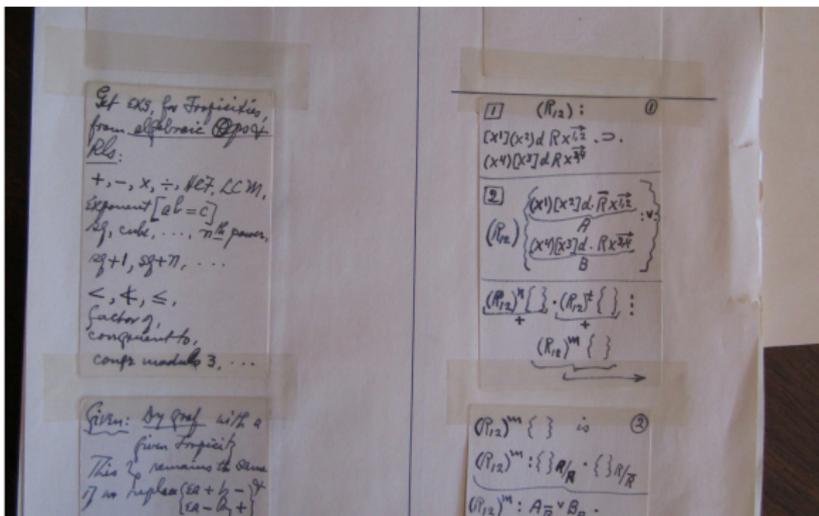
Just as, in the case of stereoscope, we have an optical Illusion consisting in the fact that one kind of spatial object (viz., the second, or surface one) parades as another (viz., as the third or solid one), so in the case of Deduction (logic) we have one kind of individual (viz., a thing ("logical fiction", like "the-so and so" or "class" or "prop (or  $\vdash$ )") parading as another kind of thing (viz., as a concept). Two kinds of individuals: (1) Things, or singular individuals (2) Concepts, or general individuals.

The Deduction Illusion (3)

Two kinds of individuals: (1) things (2 or  $\vdash$ )  
(2) concepts

Just as, in the case of the stereoscope,  
we have an optical illusion, consisting in the  
fact that one kind of spatial object (viz.,  
the second, or surface,  $\vdash$ ) parades as  
another (viz., as the third or solid,  $\vdash$ ), so

# Arrangement of Materials by H.S. Leonard, 1964



This represents only the merest beginning to a sorting out of the material here . . .

## Conclusion

- Model Theory was being developed in the 1910s by Sheffer: an algebraic, structural approach to foundations.
- The approach attempted to extract mathematical structure from *Principia* by bypassing its symbolism.
- Instead of a theory of finite strings, Sheffer tried to substitute for this a general theory of notational relativity.
- Philosophical Lesson: One cannot escape the human element in rigorizing the foundations. As Wittgenstein, Post, Church, Kleene, Turing etc. would see, the theory of definability *requires* a basis in what we do: manipulate signs. This is in fact the only way to confront conventionalism and formalism, though Sheffer didn't see this.