Against naturalism in the philosophy of mathematics

Richard Pettigrew

Department of Philosophy
University of Bristol

24th March 2011

Naturalist defences

A naturalist defence of an account of discipline X consists of:

(1) Naturalism about X The methods used by practitioners of X to justify the propositions they assert do indeed justify those propositions. This holds whether or not there is an external justification of that methodology.

(2) The account gives the correct semantics for the language of X.

(3) The account gives an accurate description of the methodology of X.

Naturalist defences

For instance, when X = mathematics:

- Burgess: the semantics of set-theoretic structuralism is correct.
- Shapiro: the semantics of *ante rem* structuralism is correct.

Thus, there are three ways to object to naturalist defences of mathematics:

- Object to Naturalism about Mathematics: this often results in stalemate.
- Object to claim to give the correct semantics for mathematics: I argue that there is no correct semantics of mathematics.
- Object to claim to have describe the methodology of mathematics accurately.

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There is no correct semantics for mathematics

What might determine the correct semantics of mathematics?

- **The intentions of mathematicians when they use mathematical language.**
- The beliefs of mathematicians that are associated with their use of mathematical language.
- **The official assertions of mathematicians.**
- The inferences that lead mathematicians to their assertions.
- The causal history of the use of mathematical language.

⋮ ⋮ ⋮ ⋮

What mathematicians intend to mean

Dedekind, *Was sind und was sollen die Zahlen?*, §73

'If in the consideration of a simply infinite system N ... we entirely neglect the special character of the elements, simply retaining their distinguishability and taking into account only the relations to one another in which they are placed ..., then are these elements called *natural numbers*.'

Penrose, *Shadows of the Mind*, 109

'The fact is ... that we actually *know* what the actual natural numbers are. The natural numbers are the ordinary things that we normally denote by the symbols 0, 1, 2, 3, 4, 5, 6, ...'

Nelson, *Predicative Arithmetic*, 2

'...numbers are symbolic constructions; a construction does not exist until it is made...'

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What mathematicians say in their official statements

How does this determine the correct semantics?

- The surface grammar determines some features.
  - For instance, singular terms are interpreted as referring uniquely.
- The *principle of minimalism* determines the rest.
  - Minimalism: Mathematical entities lack any properties that mathematicians do not themselves ascribe to them in their official statements.

Surface grammar doesn't help

The surface grammar does not determine whether 'the natural numbers' is a *singular term* or a *parameter*.

- Singular term: 'Cauchy' in 'Cauchy helped create complex analysis.'
- Parameter: 'z' in 'Let z be a complex number.'

Surface grammar doesn't help

Argument from 'Platonism and Aristotelianism in Mathematics':

(1) The formal analogues of singular terms and parameters are *constants* and *free variables*.

(2) The syntax of a formal system does not determine which expressions are constants and which free variables:
   - The same rules of inference hold of both.
   - The same rules for forming wffs hold of both.

(3) Therefore, the surface grammar of mathematics does not determine which expressions are singular terms and which parameters.

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Surface grammar doesn't help

Objection:

- The analogy is bad.
- There is more that could do the determining in the case of mathematical language than in the case of a formal language.
- Mathematical language is part of our total language.
- The correct interpretation of 'the natural numbers' might be determined by syntactic features of that term **together with facts about the correct interpretation of terms in our everyday language with those features**.

Surface grammar doesn't help

Reply:

- Why think that everyday language and technical languages function syntactically in exactly the same way?
- For instance, certain abbreviating conventions may be in place in technical language that are absent from everyday language.

Minimalism lacks motivation

Minimalism: Mathematical entities lack any properties that mathematicians do not themselves ascribe to them in their official statements.

- Consider an analogous principle for botany.
- It entails, for instance, that botanists study entities called plants that are not composed of fundamental particles.
- We should not expect an expert in discipline X to be expert in *all* features of the entities studied by X.

A more general argument?

Naturalist defences of accounts of mathematics do not work.

- They must appeal to the correct interpretation of mathematics.
- But there is no such thing.

Can a similar argument be run against naturalist defences of accounts of other disciplines?

- For instance, realism in the philosophy of science.