Philosophy Preliminary Exam
Formal Theories of Truth

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The logico-mathematical approach to understanding truth and thwarting the semantic paradoxes is largely derivative on Tarski [T35] and Kripke [K75]. In his seminal essay, Tarski proposed an object-language/metalanguage divide and showed us how to give a rigorous definition of truth for formal languages that escaped the Liar by ascending to a higher level of description. Forty years later, Kripke provided a type-free alternative to Tarski's hierarchy of languages and truth predicates by demonstrating how to define truth within the object language itself. In Kripke's fixed-point construction drawing on Kleene's strong 3-valued logic, paradoxical sentences such as the Liar lacked truth values altogether. The first part of my examination will review these foundational papers and subsequent truth theoretic constructions (see [HH02], [L07], and [M91]).

The rest of my prelim is concerned with two points of intersection between formal work on truth and more mainstream philosophical ideas on truth and logic. The first contact is the 'conservation argument' of Shapiro [S98/02] and Ketland [K99]. Given certain reasonable requirements on a truth theory T, it can be shown that $PA \cup T$ proves the consistency of PA which, by Gödel's Second Incompleteness Theorem, cannot be proven within PA itself. This presents a problem for deflationists who believe that truth has no nature; roughly, that asserting 'snow is white' is true is equivalent to asserting 'snow is white'. Deflationists must explain how such a *thin* notion of truth can teach us something new about numbers which we cannot learn from our standard theory of arithmetic. For responses to Shapiro and Ketland's argument, see [F99] and [T02].

The second contact is a recent investigation by Field [F06]. Considering theories which contain their own truth predicate, Field reasons that such theories could prove their own consistency by first proving that all of their theorems are true and then inferring consistency from the semantic property that no sentence and its negation can both be true. By Gödel's Second, we know this is impossible so the 'consistency argument' must break down somewhere. Field is interested in exactly how the argument fails in different formal theories, analyzing how the inner/outer logics of these theories differ. Given that our best formal truth theories use axioms they take to be false (*e.g.*, Kripke-Feferman) or rules of inference they do not take to be unrestrictedly truth preserving (*e.g.*, dialetheic and paracomplete theories), Field also argues against the common perception of a *valid* inference as one which necessarily preserves truth.

References :

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