TRUTH AND MEANING

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INTRODUCTION

PREAMBLE

- ***The theory outlined in my lecture, call it RCT for short, is a departure from traditional theories of truth and meaning, principally correspondence theory, coherence theory, possible-world semantics and truth-conditional semantics.***

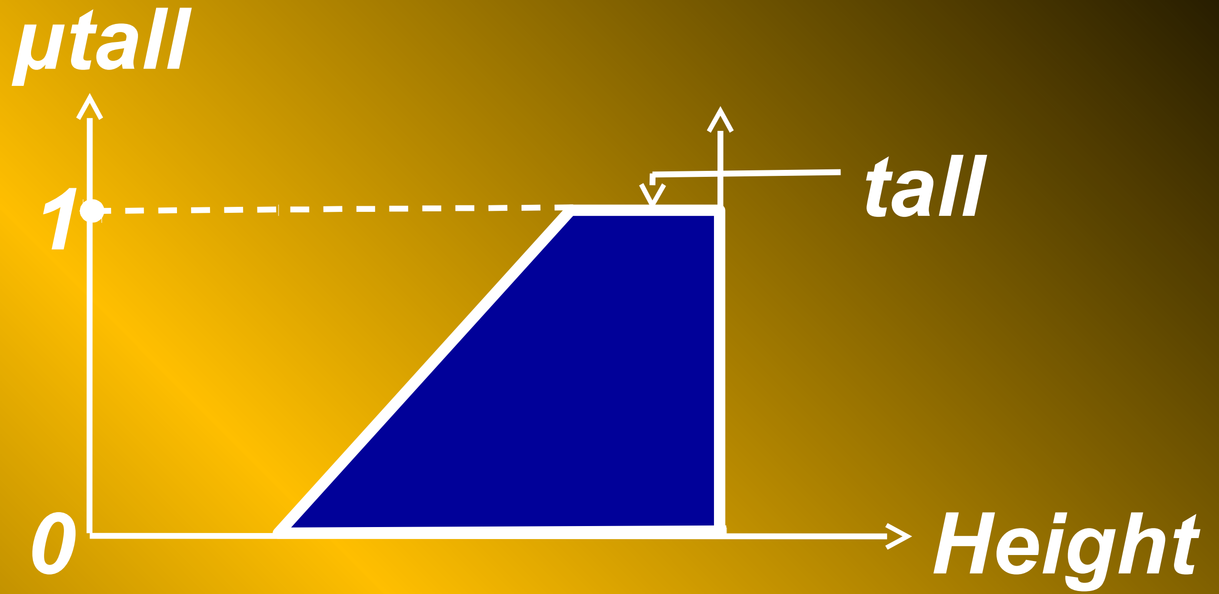
KEY POINTS

- ***The principal objective of RCT is construction of a procedure which on application to a proposition, p, drawn from a natural language leads to: (a) a mathematically well-defined (precisiated) meaning of p; and (b) truth value of p.***

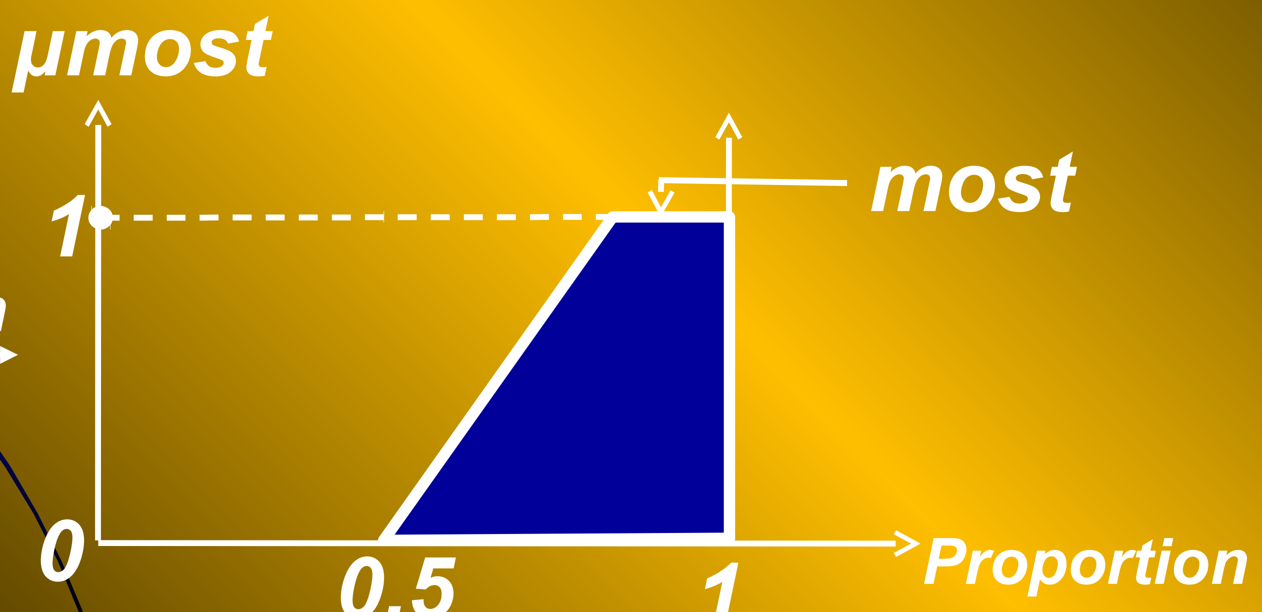
SIMPLE EXAMPLE

PRECISIATION OF CONSTITUENTS

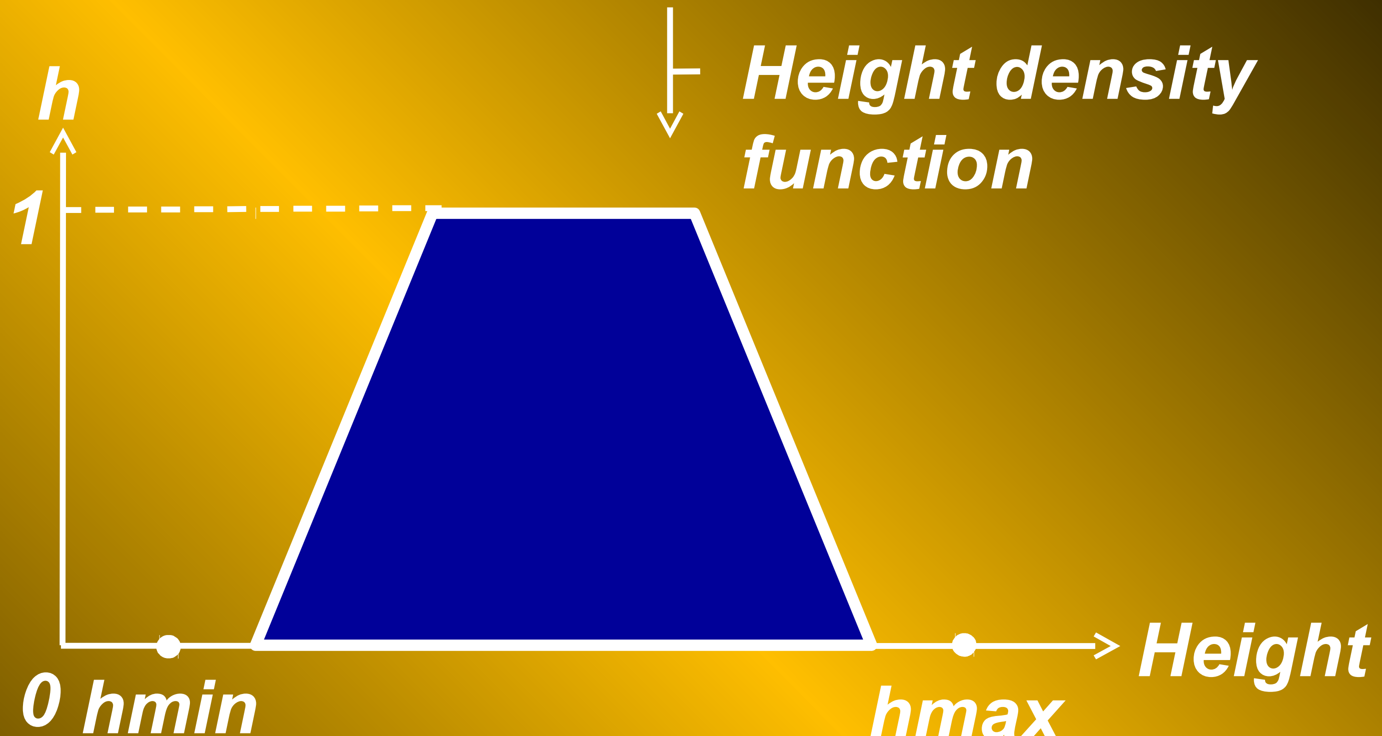
tall $\xrightarrow{\textit{precisiation}}$



most $\xrightarrow{\textit{precisiation}}$



DISTRIBUTION OF HEIGHTS



h(u)du= proportion of Swedes whose height is in the interval [u,u+du]

PRINCIPLES OF PRECISIATION

- ***In Frege's principle of compositionality, the meaning of a proposition, p, is composed from the meanings of constituents of p.***
- ***In RCT, precisiated meaning of p is composed from relations in an explanatory database, ED.***

PRECISIATION

- ***Representation of meaning is a traditional issue in semantics of natural languages.***
- ***Precisiation of meaning is not a traditional issue. Precisiation of meaning goes beyond representation of meaning.***

PRECISIATION VS. REPRESENTATION

PRECISIATION=BRIDGE

- ***Precisiation serves as a bridge between natural languages and mathematics.***

TRUTH VALUE

TWO KEY FEATURES OF RCT

- ***In RCT, truth values are numerical, e.g., 0.9, 0.2, etc., or linguistic, e.g., very true, more or less true, usually true, possibly true, etc.***
- ***In RCT, a proposition, p, is associated with two truth values—internal truth value and external truth value.***

INTERNAL AND EXTERNAL TRUTH VALUES

- Informally, the internal truth value modifies the meaning of p. The external truth value relates to the degree of agreement of p with factual information.
- A truth-qualified proposition, e.g., it is quite true that Robert is rich, is ambiguous. The meaning of p depends on whether quite true is an internal or external truth value.

THE CONCEPT OF A RESTRICTION

THE CONCEPT OF A RESTRICTION

The centerpiece of RCT is the concept of a restriction. Informally, a restriction, R(X), on a variable, X, is a limitation on the values which X can take. Typically, a restriction is described in a natural language. Simple example. Usually X is large. The concept of a restriction is more general than the concept of a constraint.

CANONICAL FORM OF A RESTRICTION

- *The canonical form of a restriction is expressed as*

R(X): X isr R,

where X is the restricted variable, R is the restricting relation and r is an indexical variable which defines the way R restricts X.

- *Note. In the sequel, the term restriction is sometimes applied to R.*

SINGULAR RESTRICTIONS

- *There are many types of restrictions. A restriction is singular if R is a singleton. Example. X=5. A restriction is nonsingular if R is not a singleton. Nonsingularity implies uncertainty.*

INDIRECT RESTRICTIONS

- *A restriction is direct if the restricted variable is X. A restriction is indirect if the restricted variable is of the form f(X).*

is an indirect restriction on p.

PRINCIPAL TYPES OF RESTRICTIONS

- ***The principal types of restrictions are: possibilistic restrictions, probabilistic restrictions and Z-restrictions.***

POSSIBILISTIC RESTRICTION (r=blank)

R(X): X is A,

where A is a fuzzy set in a space, U, with the membership function, μA. A plays the role of the possibility distribution of X,

$$Poss(X=u)=\mu A(u).$$

EXAMPLE—POSSIBILISTIC RESTRICTION

X is small

restricted variable — *restricting relation (fuzzy set)*

- ***The fuzzy set small plays the role of the possibility distribution on X.***

PROBABILISTIC RESTRICTION (r=p)

R(X): X isp p,

where p is the probability density function of X,

$$Prob(u \le X \le u + du) = p(u)du.$$

EXAMPLE—PROBABILISTIC RESTRICTION

Z-RESTRICTION (r=z, s is suppressed)

- *X is a real-valued random variable.*
- *A Z-restriction is expressed as*

R(X): X iz Z,

where Z is a combination of possibilistic and probabilistic restrictions defined as

Z: Prob(X is A) is B,

Z-VALUATIONS

- ***A Z-valuation is an ordered triple of the form (X,A,B). Equivalently, a Z-valuation, (X,A,B), is a Z-restriction on X,***

$$(X,A,B) \longrightarrow X \text{ iz } (A,B).$$

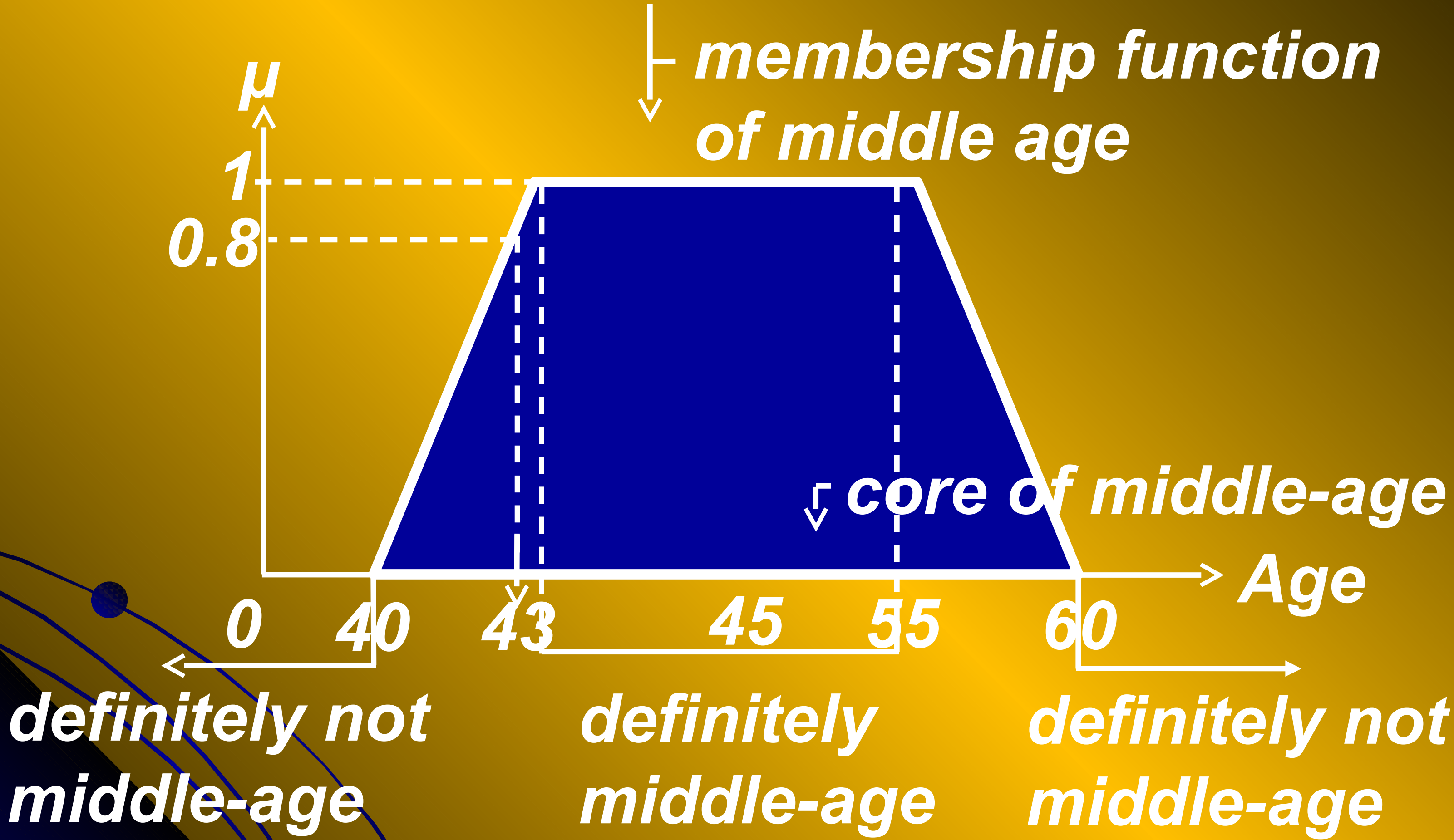
Examples.

- ***(Age(Robert), young, very likely)***
- ***(Traffic, heavy, usually).***

NATURAL LANGUAGE ≈ SYSTEM OF POSSIBILISTIC RESTRICTIONS

- ***A natural language may be viewed as a system of restrictions. In the realm of natural languages, restrictions are predominantly possibilistic. For simplicity, restrictions may be assumed to be trapezoidal.***

TRAPEZOIDAL POSSIBILISTIC RESTRICTION ON AGE



- *Note. Parameters are context-dependent.*

COMPUTATION WITH RESTRICTIONS

- ***Computation with restrictions plays an essential role in RCT. In large measure, computation with restrictions involves the use of the extension principle (Zadeh 1965, 1975). A brief exposition of the extension principle is presented in the following. The extension principle is not a single principle.***

EXTENSION PRINCIPLE—BASIC VERSION

- ***The extension principle is a collection of computational rules in which the objects of computation are various types of restrictions. More concretely, assume that Y is a function of X, Y=f(X), where X may be an n-ary variable.***

EXTENSION PRINCIPLE—A VARIATIONAL PROBLEM

Y = f(X)

R(X): X is A

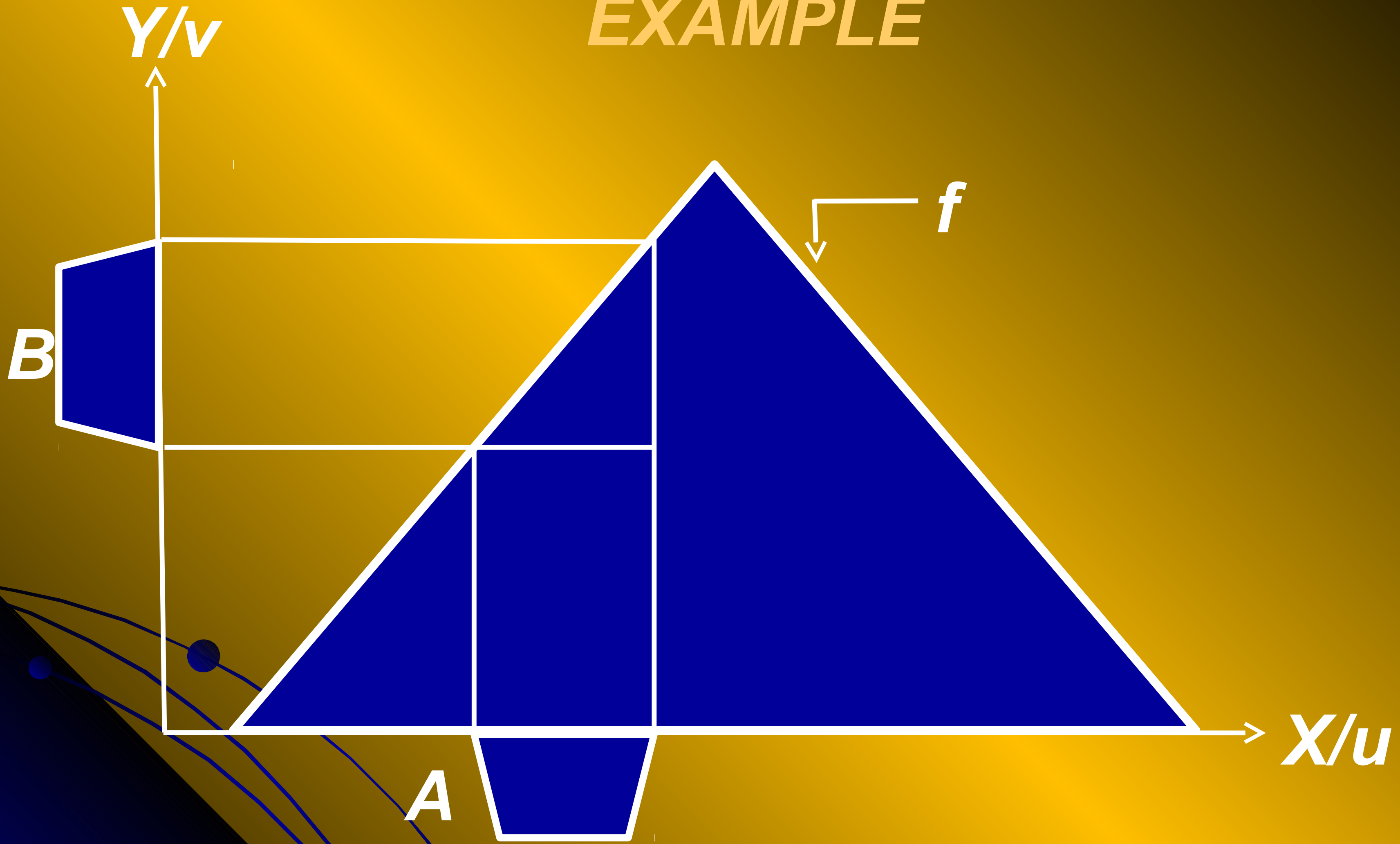
R(Y): μY(v) = supu(μA(u))

subject to

v = f(u),

where μA and μY are the membership functions of A and Y, respectively.

EXAMPLE



B = image of A under F.

INVERSE VERSION

- ***An inverse version of this version of the extension principle is the following.***

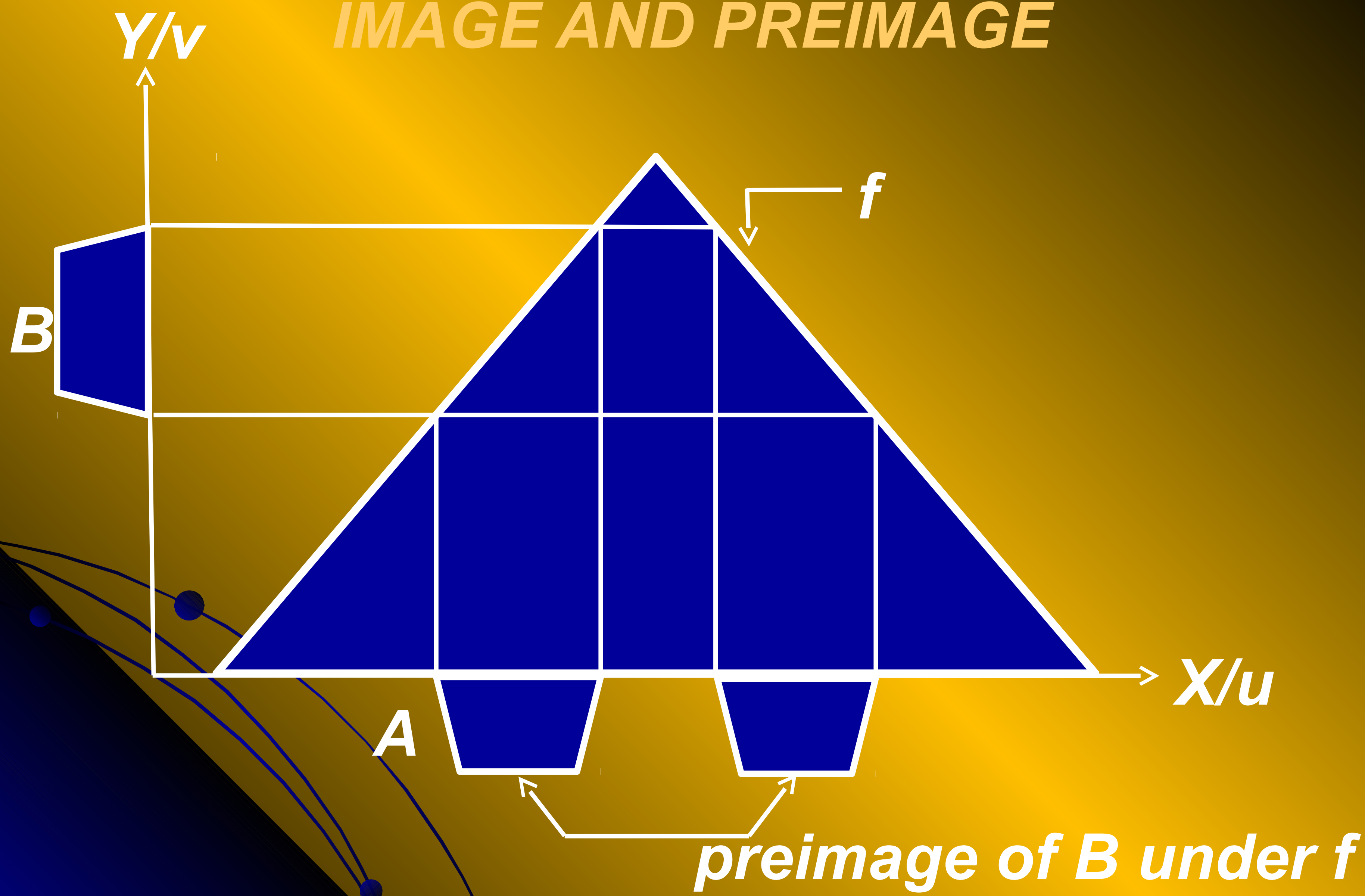
$Y = f(X)$

$R(Y)$: Y is B

$R(X)$: $\mu A(u) = (\mu B(f(u)))$

- ***Simply stated, A is the preimage of B under f.***

IMAGE AND PREIMAGE



TRUTH AND MEANING

TRUTH AND MEANING

- ***To assess the truth value of a proposition, p, it is necessary to understand the meaning of p. However, understanding the meaning of p is not sufficient. What is needed, in addition, is precisiation of the meaning of p.***

PRECISIATION OF MEANING

- ***Precisiation of the meaning of p involves representation of p in a form that is mathematically well defined and lends itself to computation. In RCT, formalization of the concept of truth is a byproduct of formalization of the concept of meaning.***

FUZZY PROPOSITIONS

- ***In the following, p is assumed to be a proposition drawn from a natural language. Typically, propositions drawn from a natural language are fuzzy propositions, that is, propositions which contain fuzzy predicates and/or fuzzy quantifiers and/or fuzzy probabilities.***
- ***Example. p: It is very unlikely that there will be a significant decrease in the price of oil in the near future.***

MEANING POSTULATE, MP AND TRUTH POSTULATE, TP

- ***The point of departure in RCT consists of two key ideas:***

  MP=meaning postulate;

  TP=truth postulate.

- ***MP: p $\longrightarrow$ restriction $\longrightarrow$ X isr R,***
- ***TP: Tr(p)=truth value of p***
- ***Tr(p)=degree to which X satisfies R***

EXPLICITATION OF X and R

- X, R and r are implicit in p. The expression X isr R is referred to as the canonical form of p, CF(p). Basically, X is a variable such that p is a carrier of information about X. X is referred to as a focal variable of p. In general, X is an n-ary variable and R is a function of X.

EXAMPLES

p: Robert is young $\rightarrow$ *Age(Robert) is young*

$\uparrow$ $\uparrow$

X *R*

p: Most Swedes are tall $\rightarrow$

Proportion(tall Swedes/Swedes) is most

$\uparrow$ $\uparrow$

X *R*

p: Robert is much taller than most of his friends $\rightarrow$ *Height(Robert) is much taller than heights of most of his friends.*

FACETS OF TRUTH VALUES

- ***A generic numerical truth value is denoted as nt. nt takes values in the unit interval. In RCT, a linguistic truth value, lt, is interpreted as a possibilistic restriction on numerical truth values, implying that lt is a fuzzy set. In symbols, lt=R(nt). A generic truth value is denoted as t. t may be nt or lt.***

EXPLANATORY DATABASE (ED)

- *Typically, X and R are described in a natural language. To compute the degree to which X satisfies R, it is necessary to precisiate X and R. In RCT, what is used for this purpose is the concept of an explanatory database, ED (Zadeh 1983, 2012)*

PRECISIATION OF X, R AND p

- *Informally, ED is a collection of relations which represent the information which is needed to precisiate X and R or, alternatively, to compute the truth value of p. Example.*

p: Most Swedes are tall.

EXPLANATORY DATABASE (ED)

p: Most Swedes are tall

ED →

TALL	Height	μ

MOST	Proportion	μ

POPULATION	Name	Height

- ***Note. In instantiated ED what are instantiatied are the constituent relations.***

KEY POINTS

- ***Truth and meaning are closely related.***
- ***ED=information which is needed to compute the truth value of p.***
- ***Truth=degree of agreement of meaning of p and ED.***
- ***Meaning=procedure for computing the truth value of p.***

MOST SWEDES ARE TALL

- ***In this case, the information consists of three relations, TALL[Height;μ], MOST[Proportion;μ] and POPULATION[Name;Height]. In TALL, μ is the grade of membership of Height in tall.***

EXPLANATORY DATABASE: µtall, µmost, and h

- Equivalently, and more simply, ED may be taken to consist of the membership function of tall, µtall, the membership function of most, µmost, and the height density function, h. h is defined as the fraction, h(u)du, of Swedes whose height is in the interval [u,u+du].

PRECISIATION OF X, R AND p

- X and R are precisiated by expressing them as functions of ED. Precisiated X, R and p are denoted as X*, R* and p*, respectively. Thus,

$$X^* = f(ED) \;,\; R^* = g(ED).$$

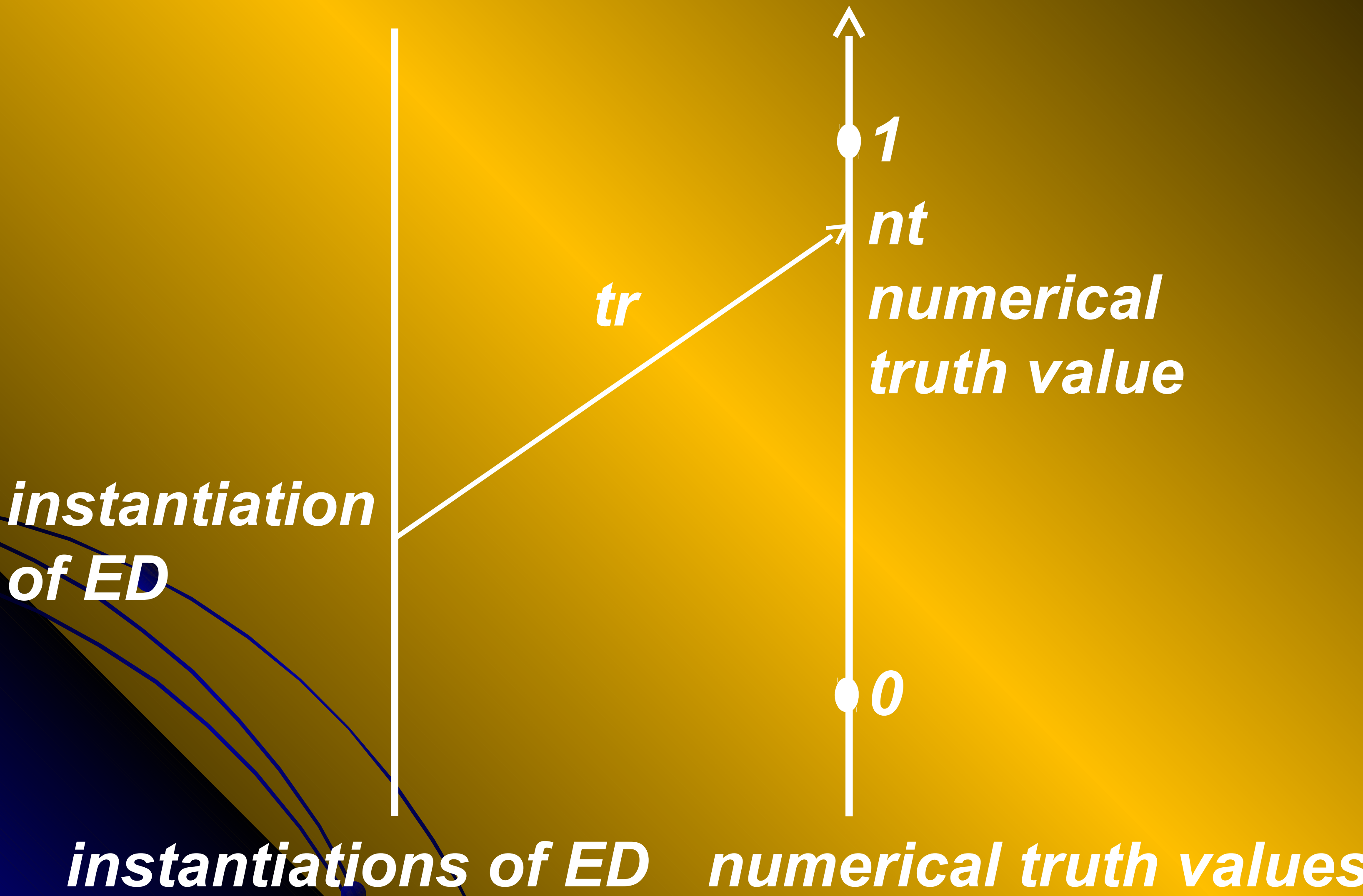
PRECISIATED CANONICAL FORM

- The precisiated canonical form, CF*(p), is expressed as X*isr* R*. At this point, the numerical truth value of p, ntp, may be computed as the degree to which X* satisfies R*. In symbols,

  ntp = tr(ED),

  in which tr is referred to as the truth function.

TRUTH FUNCTION



TRUTH DISTRIBUTION OF p

- ***What this equation means is that an instantiation of ED induces a value of ntp. Varying instantiations of ED induces what is referred to as the truth distribution of p, denoted as Tr(p|ED). The truth distribution of p may be interpreted as the possibility distribution of ED given p, expressed as Poss(ED|p).***

BASIC EQUALITY

$$Tr(p|ED) = Poss(ED|p).$$

- *In RCT, the precisiated meaning of p is expressed in three equivalent forms. First, as the precisiated canonical form, CF*(p).*

PRECISIATED MEANING OF p=COMPUTATION MEANING OF p

- ***Second, as the truth distribution of p, Tr(p|ED). Third, as the possibility distribution, Poss(ED|p). These representations of the precisiated meaning of p play an essential role in RCT. The precisiated meaning of p may be viewed as the computational meaning of p.***

PRECISIATED MEANING OF p=POSS(ED|p)

- Of the three equivalent definitions stated above, the definition that is best suited for computational purposes is that which involves the possibility distribution of ED. Adopting this definition, what can be stated is the following.

DEFINITION OF MEANING

- ***Precisiated (computational) meaning of p is the possibility distribution of ED, Poss(ED|p), which is induced by p.***

EXAMPLE

- ***Consider the proposition, p: Most Swedes are tall. In this case, X=Prop(tall Swedes/Swedes) and R=most. The canonical form of p is***

Prop(tall Swedes/Swedes) is most.

EXAMPLE (CONTINUED)

EXAMPLE (CONTINUED)

EXAMPLE (CONTINUED)

$$Tr(p|ED)=\mu_{most}\left(\int_{h_{min}}^{h_{max}} h(u)\mu_{tall}du\right).$$

RELATION TO THE CONCEPTS OF POSSIBLE WORLD AND INTENSION

- ***Note. The concept of an instantiated ED in RCT is related to the concept of a possible world in traditional theories. Similarly, the concept of a truth distribution of p is related to the concept of intension of p.***

FROM p TO PRECISIATION OF p

- Precisiation of meaning is the core of RCT and one of its principal contributions. A summary may be helpful.

1. Identify X and R.
2. Construct the canonical form, CF(p): X isr R.
3. Construct the explanatory database, ED.

FROM p TO PRECISIATION OF p

4. ***Precisiate X and R—X* and R*.***
5. ***Construct the precisiated canonical form, CF*(p): X* isr R*.***
6. ***Form the possibility distribution, Poss(ED|p).***
7. ***Precisiated p=Poss(ED|p).***

QED

INTERNAL AND EXTERNAL TRUTH VALUES

- *In a departure from tradition, in RCT a proposition, p, is associated with two truth values—internal truth value and external truth value. When necessary, internal and external truth values are expressed as Int(t) and Ext(t).*

DEFINITION OF INTERNAL TRUTH VALUE

- ***Informally, the internal numerical truth value is defined as the degree of agreement of p with an instantiation of ED. Informally, an external numerical truth value of p is defined as the degree of agreement of p with factual information, F. More concretely, an internal numerical truth value is defined as follows.***

DEFINITION OF INTERNAL TRUTH VALUE

Int(ntp) = tr(ED)

- ***In this equation, ED is an instantiation of the explanatory database, Int(ntp) is the internal numerical truth value of p, and tr is the truth function which was defined earlier.***
- ***The definition of truth value which was given earlier is that of an internal truth value.***

POSSIBILISTIC RESTRICTION ON ntp

- *More generally, assume that we have a possibilistic restriction on instantiations of ED, Poss(ED). This restriction induces a possibilistic restriction on ntp which can be computed through the use of the extension principle. The restriction on ntp may be expressed as tr(Poss(ED)).*

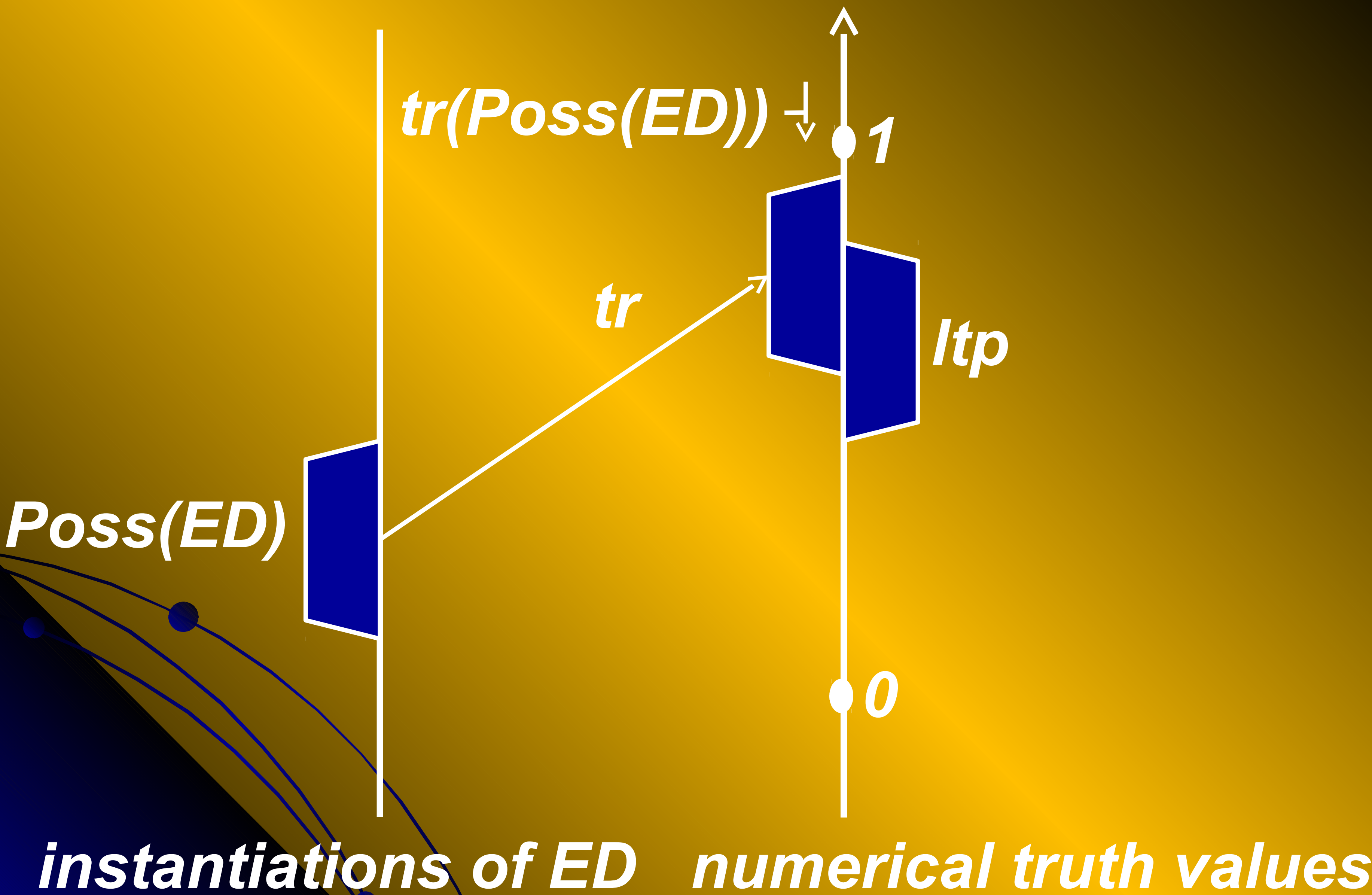
DEFINITION OF INTERNAL LINGUISTIC TRUTH VALUE

- ***The fuzzy set, tr(Poss(ED)), may be approximated by the membership function of a linguistic truth value. This leads to the following definition of an internal linguistic truth value of p.***

DEFINITION

$$Int(Itp) \approx tr(Poss(ED)).$$

- ***In this equation, ≈ should be interpreted as a linguistic approximation. In words, the internal linguistic truth value, Int(Itp), is the image—modulo linguistic approximation—of the possibility distribution of ED under the truth function, tr.***


tr(Poss(ED))
1
tr
ltp
Poss(ED)
0
instantiations of ED
numerical truth values

EXTERNAL TRUTH VALUE

- ***The external truth value of p, Ext(p), relates to the degree of agreement of p with factual information, F. In RCT, factual information is assumed to induce a possibilistic restriction on ED, Poss(ED|F). In particular, if F instantiates ED, then the external truth value is numerical. This is the basis for the following definition.***

DEFINITION OF EXTERNAL TRUTH VALUE

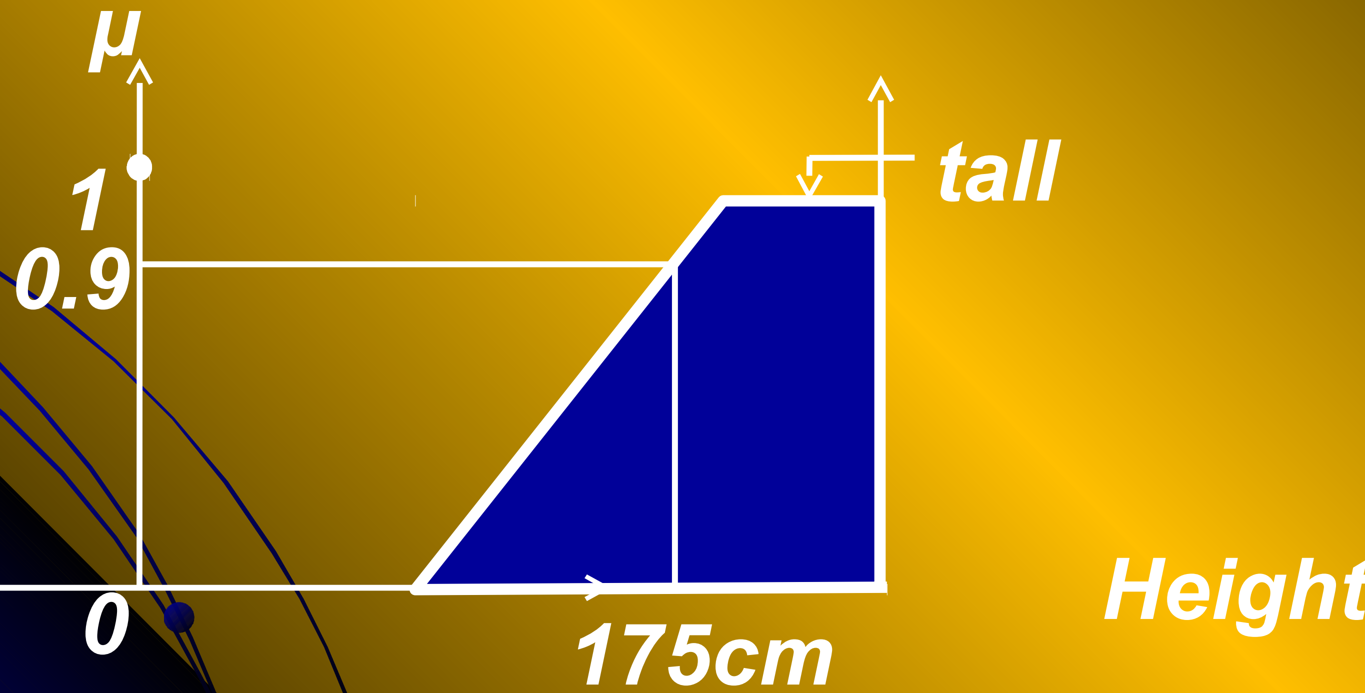
The external numerical truth value of p is defined as

$$Ext(ntp) = tr(ED|F),$$

where ED is an instantiation of the explanatory database induced by F.

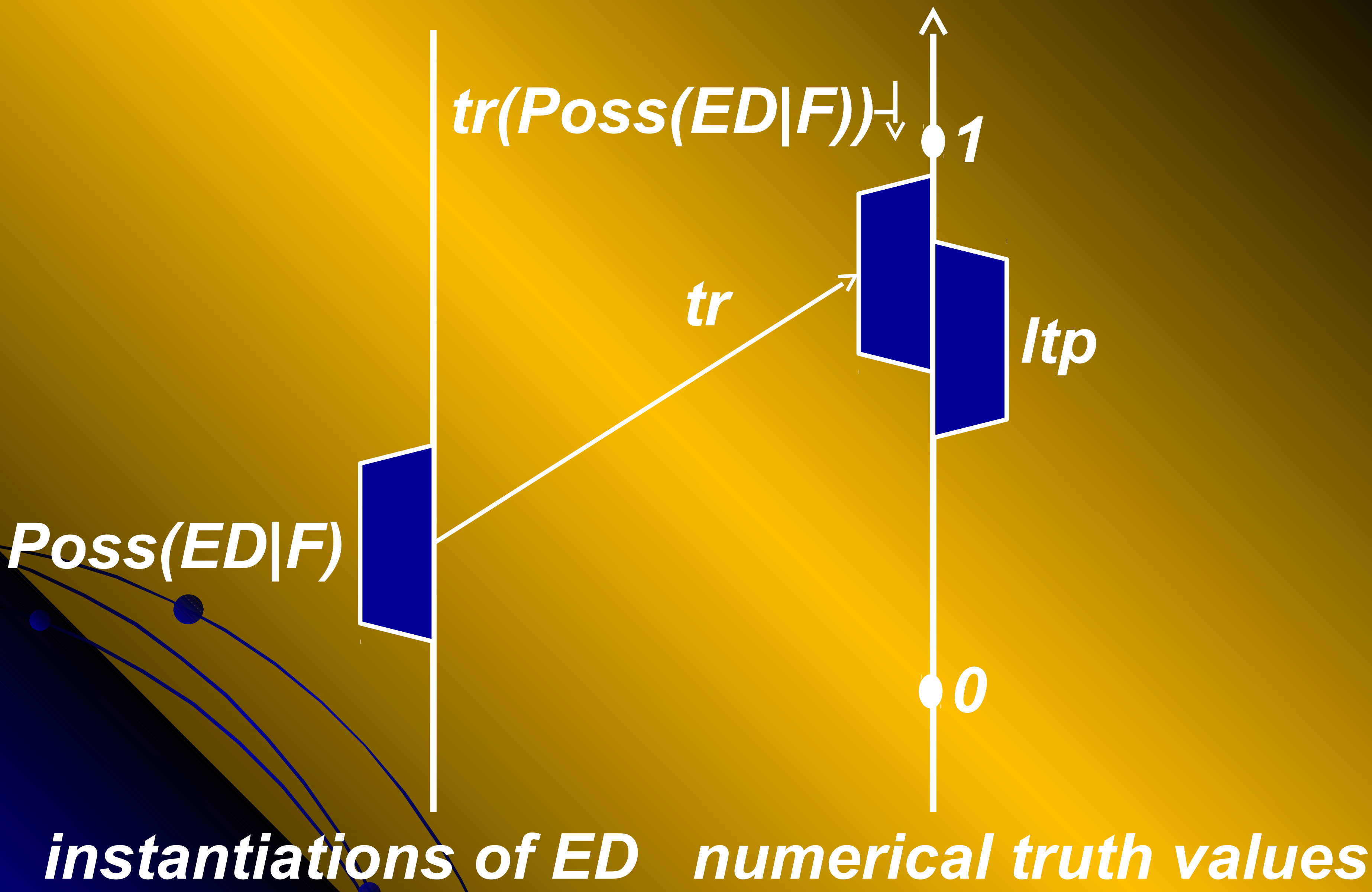
SIMPLE EXAMPLE

- ***If the factual information is that Robert's height is 175cm, then the external numerical truth value of p is 0.9.***



SIMPLE EXAMPLE (CONTINUED)

- *More generally, if F induces a possibilistic restriction on instantiations of ED, Poss(ED|F), then the external linguistic truth value of p may be defined as shown in the next slide.*



tr(Poss(ED|F))
1
tr
ltp
Poss(ED|F)
0
instantiations of ED
numerical truth values

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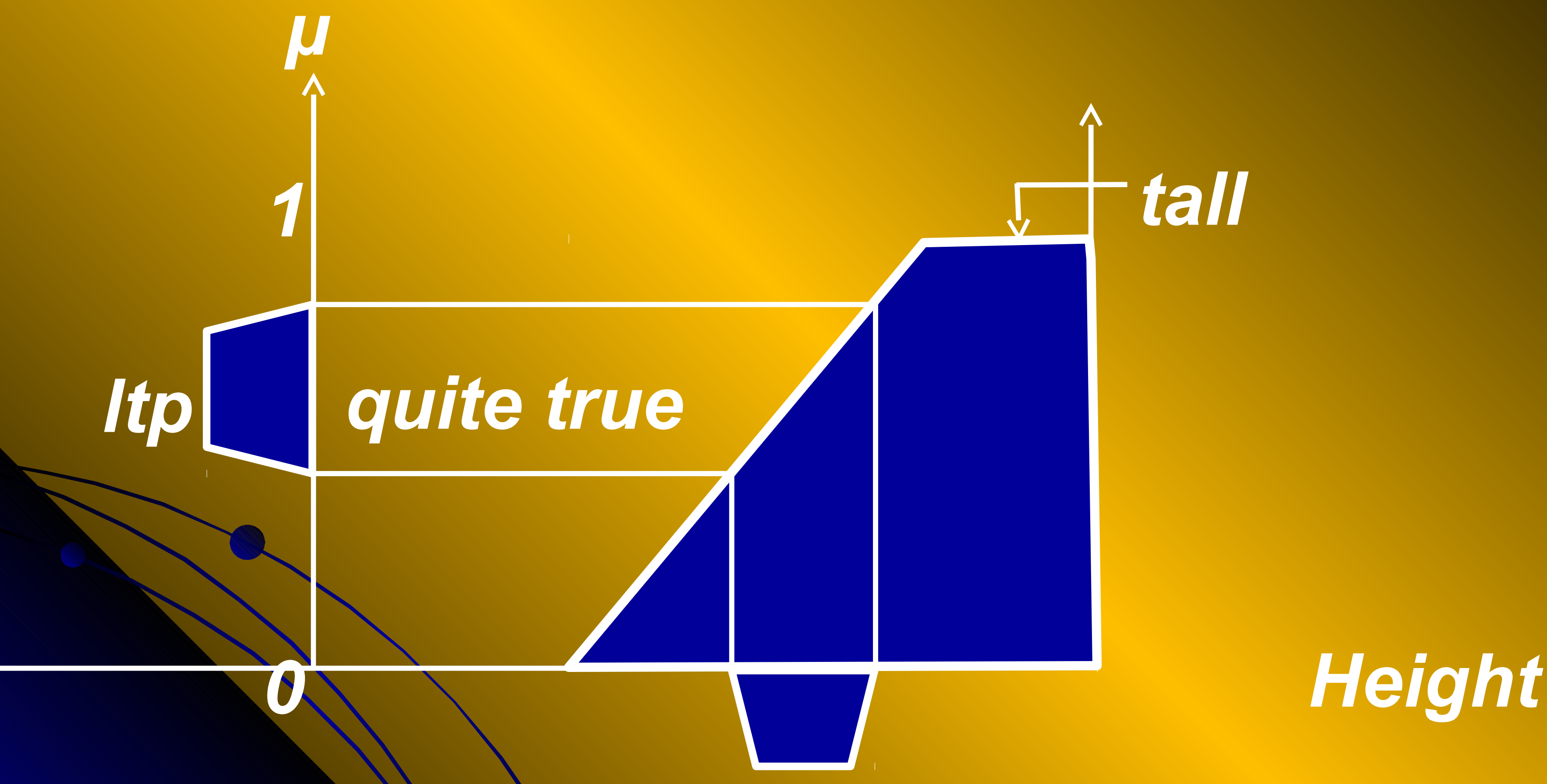
TRUTH QUALIFICATION

- *A truth-qualified proposition is a proposition of the form t p, where t is the truth value of p. t may be a numerical truth value, nt, or a linguistic truth value, lt. Example. It is quite true that Robert is tall. In this case, t=quite true and p=Robert is tall.*

TRUTH QUALIFICATION—MODIFICATION OF MEANING VIA INTERNAL TRUTH VALUE



preimage of ltp(modified meaning of p)

MODIFICATION OF MEANING BY HEDGED TRUTH VALUE—A SPECIAL CASE

- ***There is a special case which lends itself to a simple analysis. Assume that lt is of the form h_true, where h is a hedge exemplified by quite, very, almost, etc. Assume that p is of the form X is A, where A is a fuzzy set. In this case, what can be postulated is that truth-qualification modifies the meaning of p as follows:***

SIMPLE EXAMPLE

h_true(X is A) = X is h_A.

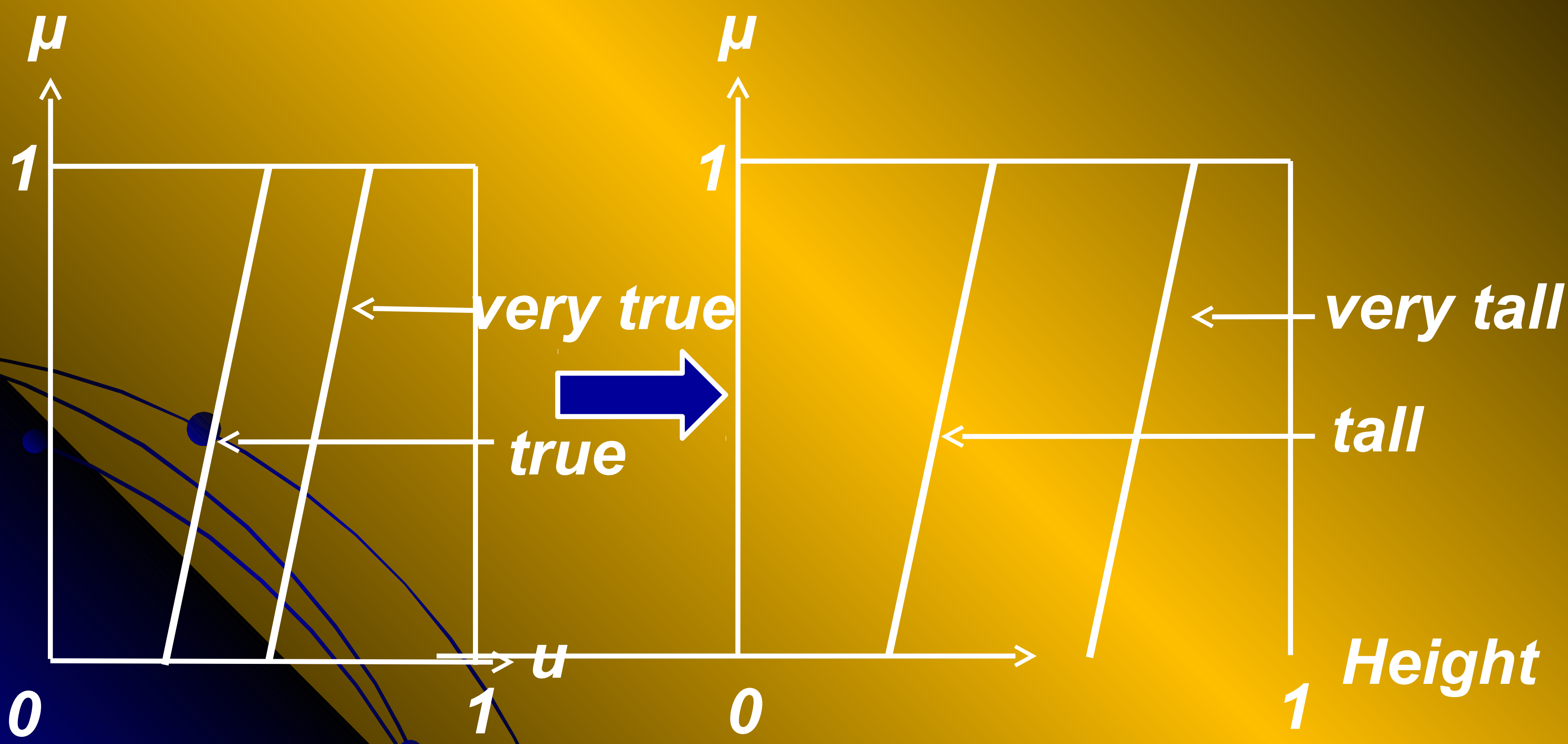
- ***h_A may be computed through the use of techniques described in early papers on hedges (Zadeh 1972, Lakoff 1972)***

Example.

(usually_true) snow is white = snow is usually white.

SIMPLE EXAMPLE

It is very true that Robert is tall = Robert is very tall.



CONCLUDING REMARK

- ***The theory outlined in this paper, RCT, serves as a bridge between natural language and mathematics.***
- ***RCT opens the door to construction of mathematical solutions of computational problems which are stated in natural language.***
- ***Traditional theories do not have this capability.***

CONCLUDING REMARK (CONTINUED)

- ***The theory which underlies RCT is not easy to understand, largely because it contains many unfamiliar concepts. However, once it is understood, what is revealed is that the conceptual structure of RCT is simple and natural.***